Optical fiber interferometry

- **Introduction**
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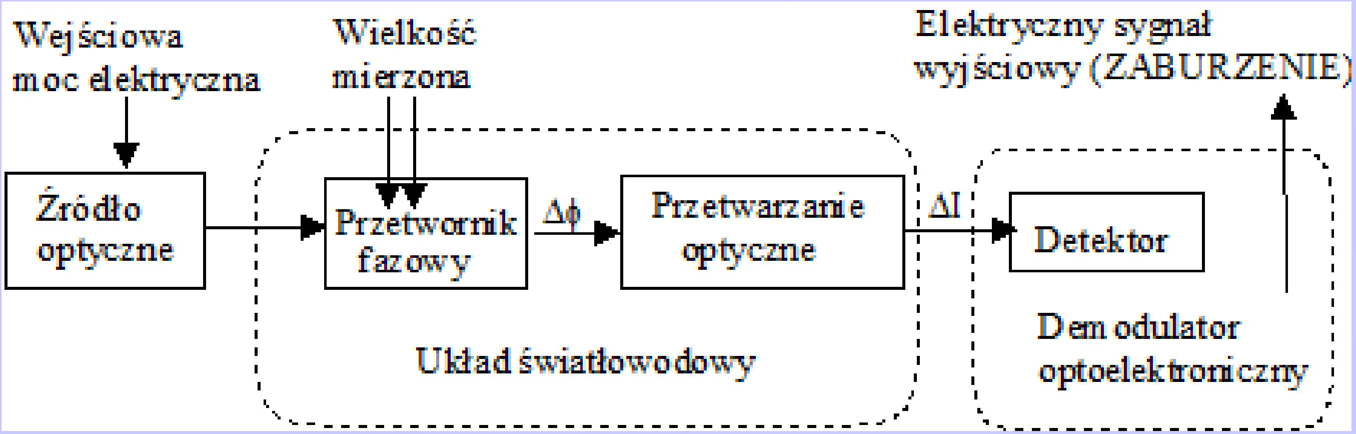
- **Signal processing**
  - **optical considerations**
  - **electronic considerations**

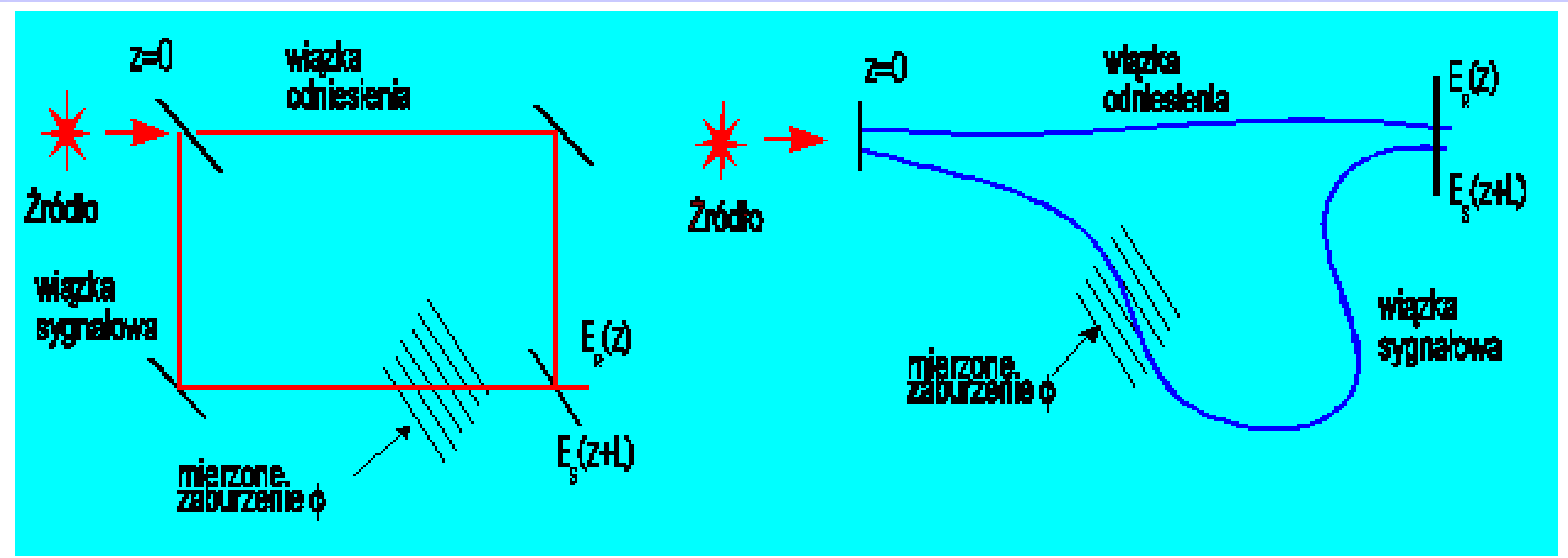
Introduction

They are a new sensor class, where general principle of operation based on chenges by measurand the optical way or polarization properties of optical fiber. Such sensors are named *intrinsic* or *phase* sensors.



For optical fiber, optical processing technique which process phase information on intensity information (ΔI), is called *optical-fber intereferometer*. It is a simple fiber-optic device allows observation of a interference between two or more optical beams.

Fiber optic interfereometers are equivalents all well-known optical bulk interfereometers, where light is closed in structure of single-mode or polarization preserving optical fiber.



General principles

An optical sensor may be formally defined as a device in which an optical signal is modulated in response to a measurand field. Let us assume that the source has some well-defined wavelength spectrum, and that the electric field at wavelength λ is $E(\lambda)$, in unit bandwidth. Then the corresponding received electric field will be:

$$\mathbf{E}'(\lambda) = \boldsymbol{T}(\mathbf{X},\lambda)\mathbf{E}(\lambda)$$

where $\boldsymbol{T}(\mathbf{X}, \lambda)$ is the propagation matrix describing the sensing element and **X** is a vector describing the physical environment, including terms representing temperature, stress, electromagnetics fields. The function of the signal processing used in the sesnor system is to invert above relation, to find ***T*,** invert again to find **X,** and then to idenftify and evaluate the reveland component(s) of **X** to recover the desired measurand.

It is instructive to express ***T*** as a product of terms, each describing a physically obsevable effect on the transmited beam, such that:

$$\boldsymbol{T} = a e^{i\phi_1} \boldsymbol{B}$$

where: *a* – scalar transmittance, ϕ_1 – the mean phase retardance, and ***B*** are all both dispersive and environmentally sensitive. In a multimode fiber, spatial coherence is not fully maintained, so that for many simple multimode fiber sensors the device is entirely characterized by the dispersicve scalar transmittance, *a*.

In modnomode systems, sensing mechanisms based on modulation of any one or combination of the parameters *a,* $\phi_{1,}$, ***B***. In practice, the transmittance of SMF shown only weak environmental sensitivity, and intrinsic monommode sensors are thus generally based on phase and polarization modulation.

The transfer function of a monommode sensing element in the Jones calculus is::

$$\mathbf{E}' = a_0 \mathbf{E} e^{i\phi_1} \boldsymbol{B}$$

where: a_0 – scalar constant (transmmittance) and ***B*** – the Jones matrix.

For a SMF possessing perfect cylindrical symmetry ***B*** = ***I***, but general it is necessary to consider the effects of birefringence within the fiber. For example for a linear birefringent fiber:

$$\boldsymbol{B} = \boldsymbol{B}_l = \begin{bmatrix} e^{i\phi_2/2} & 0 \\ 0 & e^{-i\phi_2/2} \end{bmatrix}$$

Such a fiber is characterised by two linear polarization eighenmodes, such that ϕ_2 is the induced relative phase retardance between the eigenmodes caused by propagation through the fiber. For a circularly birefringent fiber:

$$\boldsymbol{B} = \boldsymbol{B}_c = \begin{bmatrix} \cos\phi_3 & -\sin\phi_3 \\ \sin\phi_3 & \cos\phi_3 \end{bmatrix}$$

where: $2\phi_3$– the induced relative phase retardance between the eigenmodes, which in this case are left and right circularly polarized states.

The environmental sensitivity of the fiber can be described in terms of above phase ϕ_i (i=1,2,3) dependencies on external stimuli such as temperature (T), pressure (P), and strain ($\Delta l/l$):

$$\frac{\partial \phi_i}{\partial X} = \frac{2\pi}{\lambda}\left(n_i \frac{\partial l}{\partial X} + l\,\frac{\partial n_i}{\partial X}\right);\ X = \mathrm{T,P},\Delta l, \ldots;\ i = 1,2,3$$

l – the length of the fiber, n_i - relative refractive index of fiber (1- fiber, 2- difference between refractive index of two linear eigenmodes, 3 – differece of refractive index betweeen circular eigenmodes)

Table 10.1 Temperature and strain sensitivity coefficients measured using 0.1 m of highly birefringent monomode optical fiber (York Technology "bow tie" fiber, 3 mm beat length) at a wavelength of 633 nm

X	$(1/l)(\partial\phi_1/\partial X)$	$(1/l)(\partial\phi_2/\partial X)$
T	100	5 rad K^{-1} m^{-1}
$\Delta l/l$	6.5×10^6	6.5×10^4 rad m^{-1}

Transfer function:

$$I = I'\{1+V(\gamma)\cos[\phi-\alpha(\gamma)]\}$$

bulk system

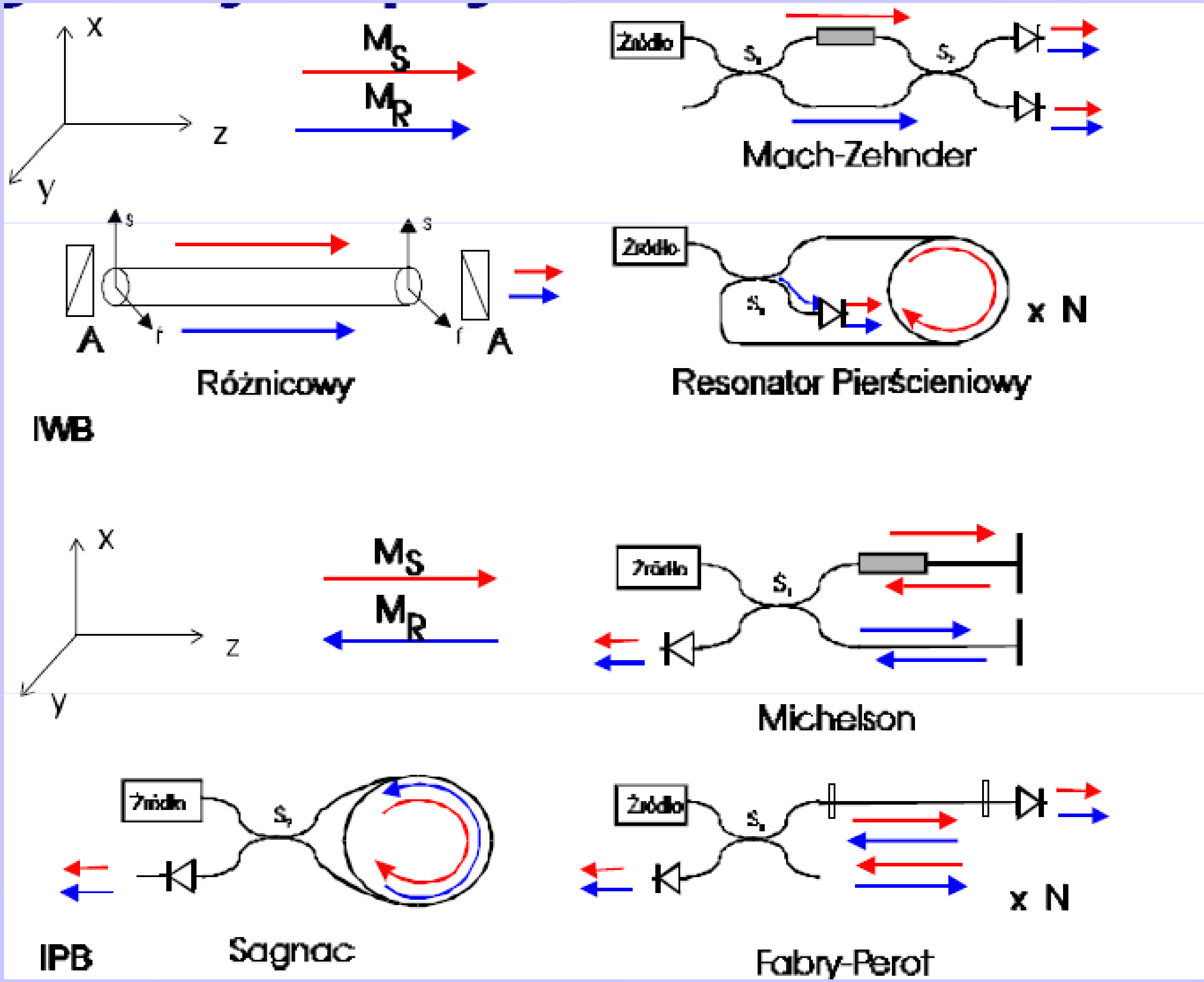
$$I = I'\{1+V(\gamma,SOP)\cos[\phi-\alpha(\gamma,SOP)]\}$$

fiber-optic system

Optical processing

The phase or polarization demodulation optical carrier is obtained by interefrometer system application, futher siganl processing on useful form is realized by electronics processing.

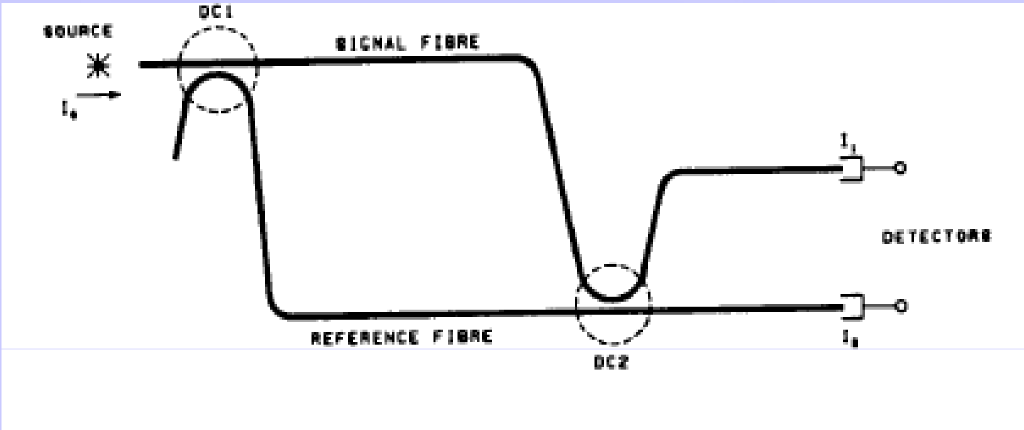
IWB (CMW) – Common–way interferometer
IPB (CCW) – Conter-way interferometer
ANALYSE:
1. Coupler influence
2. PT localisation influence
3. Configuration influence



Ideal construction (without polarisation)

$$E_1 = k_{2c}\exp(i\phi_b)k_{1c}E_0(\tau_b) + k_{2t}\exp(i\phi_a)k_{1t}E_0(\tau_a)$$

$$E_2 = k_{2t}\exp(i\phi_b)k_{1c}E_0(\tau_b) + k_{2c}\exp(i\phi_a)k_{1t}E_0(\tau_a)$$



Coupler description: $k_{1c} = ik'_{1c}$

Interference: $I_1 = \langle E_1 \cdot E_1^* \rangle$

$$I_1 = k'^2_{1c}k'^2_{2c}\langle E^2_0(\tau_b)\rangle + k^2_{1t}k^2_{2t}\langle E^2_0(\tau_a)\rangle + 2\mathrm{Re}[-k'_{1c}k'_{2c}k_{1t}k_{2t}\, e^{i(\phi_a-\phi_b)}\langle \mathbf{E}_0(\tau_a)\cdot \mathbf{E}^*_0(\tau_b)\rangle]$$

The degree of coherence of the source: $\gamma(\tau_a - \tau_b) = \langle \mathbf{E}_0(\tau_a)\cdot \mathbf{E}^*_0(\tau_b)\rangle / I_0$

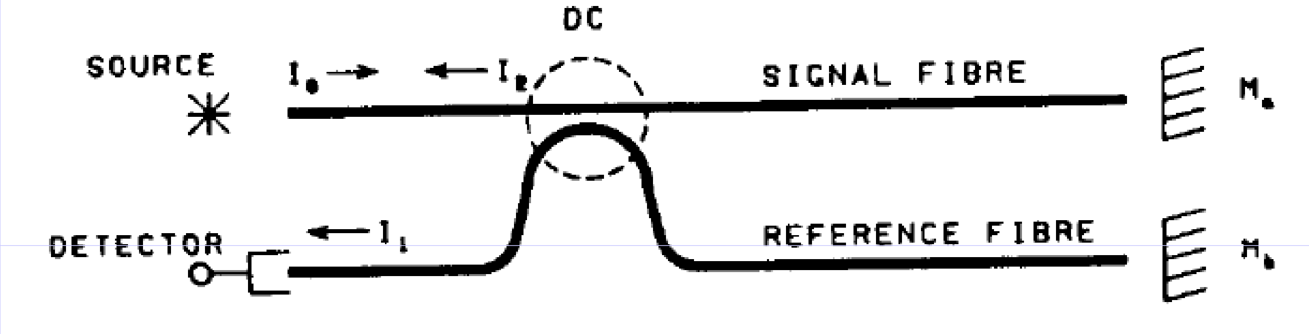
Visibility (contrast) of the interference: $V = (I_{max} - I_{min})/(I_{max} + I_{min})$

Intensity on detector: $I_1 = I_0[1 - V\cos(\phi_a - \phi_b)]$ $\quad I_2 = I_0[1 + V\cos(\phi_a - \phi_b)]$

$$V = \frac{2k_{1c}k_{2c}k_{1t}k_{2t}}{k^2_{1c}k^2_{2c} + k^2_{1t}k^2_{2t}}\gamma \qquad |\gamma(\tau)| = \lim_{T\to\infty}\frac{1}{T}\int_0^T \exp i[\phi(t+\tau) - \phi(t)]\,dt$$

Practially: $\tau_c \gg \tau_a - \tau_b$ and $k'_{ic} = k_{it} = 1/\sqrt{2}$. thus $\gamma=1$, $V=1$

Ideal construction (without polarisation)



Electrical field on detector:

$$E_1 = k_c m_a \exp(i\phi_a) k_t E_0(\tau_a) + k_t m_b \exp(i\phi_b) k_c E_0(\tau_b)$$

$$E_2 = k_t m_a \exp(i\phi_a) k_t E_0(\tau_a) + k_c m_b \exp(i\phi_b) k_c E_0(\tau_b)$$

Intensity on detector:

$$I_1 = I_0 \,[1 - V\cos(\phi_a - \phi_b)]$$

$$I_2 = I_0 \,[1 + V\cos(\phi_a - \phi_b)]$$

$$V = 2 m_a m_b \gamma / (m_a + m_b)$$

With polarisation – ideal construction: the Kapron law about an optical system equivalence:

$$\mathbf{M}_s \equiv \mathbf{R}(\Omega)\mathbf{G}(\delta)\mathbf{M}(\Phi)$$

The Jones vector for the optical source:

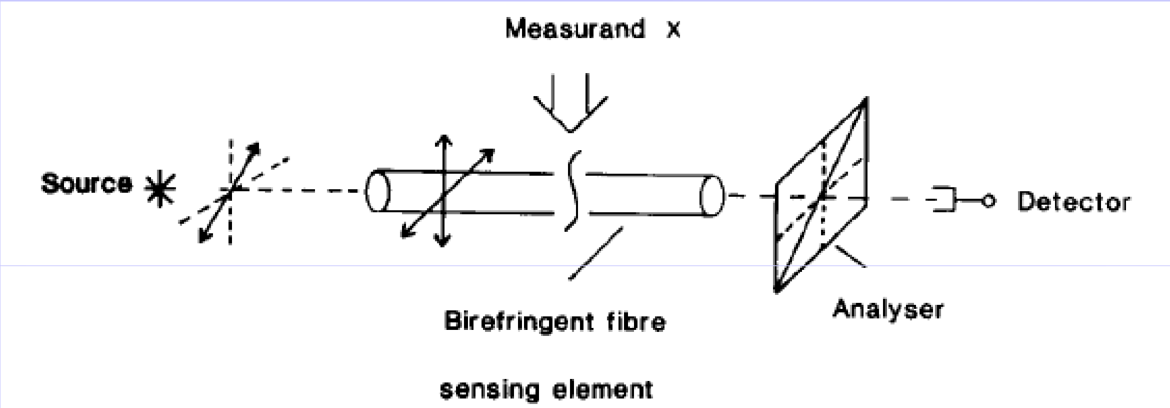
$$\mathbf{E} = e^{i\omega t}\begin{bmatrix} E_x \\ E_y e^{i\Delta} \end{bmatrix} = \ldots = \begin{bmatrix} E_x \\ E_y e^{i\Delta} \end{bmatrix} = \ldots = \begin{bmatrix} \cos\beta \\ \sin\beta e^{i\Delta} \end{bmatrix}$$

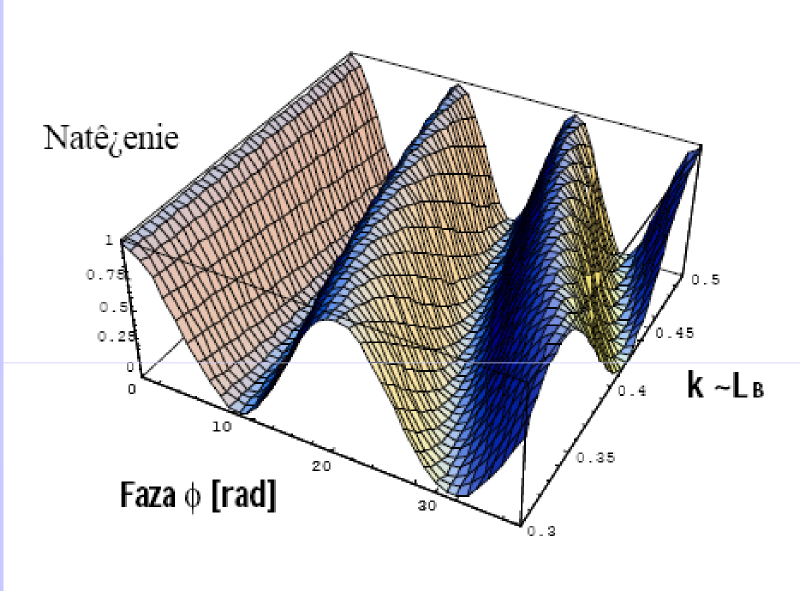
General matrix of system: $m = \mathbf{E}_{we}^{+}\mathbf{M}_{R}^{+}\mathbf{M}_{S}\mathbf{E}_{we} \in C$

Transfer function: $I = 0.5\{1 \pm V\cos[\phi' + \phi_0]\}$

V=Abs(m)	$\phi'=f(\phi)$	ϕ_0 =arg(m)
Scale factor Responsitivity	Configuration Dynamic range	Bias Drift

Linear polarimeter:





Transfer function:

$$\mathbf{E}_1 = \boldsymbol{A}(\alpha)\,\boldsymbol{F}\,\mathbf{E}_0$$

$$\boldsymbol{A} = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} e^{i(\phi_1 + 1/2\phi_2)} & 0 \\ 0 & e^{i(\phi_1 - 1/2\phi_2)} \end{pmatrix}$$

$$\mathbf{E}_0 = \begin{pmatrix} \cos\delta \\ e^{i\beta}\sin\delta \end{pmatrix}$$

$$I = \frac{1}{2} I_0 (1 + \cos 2\alpha \cos 2\delta)[1 + V\cos(\phi_2 - \beta)]$$

$$V = \sin 2\alpha \sin 2\delta/(1 + \cos 2\alpha \cos 2\delta)$$

for $\delta=\alpha=\pi/4$

$$I_1 = \frac{1}{2} I_0 [1 + \cos(\phi_2 - \beta)]$$

The polarisation properties change has influence only on dynamic range

Circular polarimeter:

$$\boldsymbol{F} = \begin{pmatrix} \cos\phi_3 & \sin\phi_3 \\ -\sin\phi_3 & \cos\phi_3 \end{pmatrix} \quad \mathbf{E}_0 = \begin{bmatrix} \cos\delta \\ \sin\delta \end{bmatrix}$$

$$\Rightarrow \quad I_1 = \frac{1}{2} I_0[1 + \cos 2(\phi_3 + \delta - \alpha)]$$

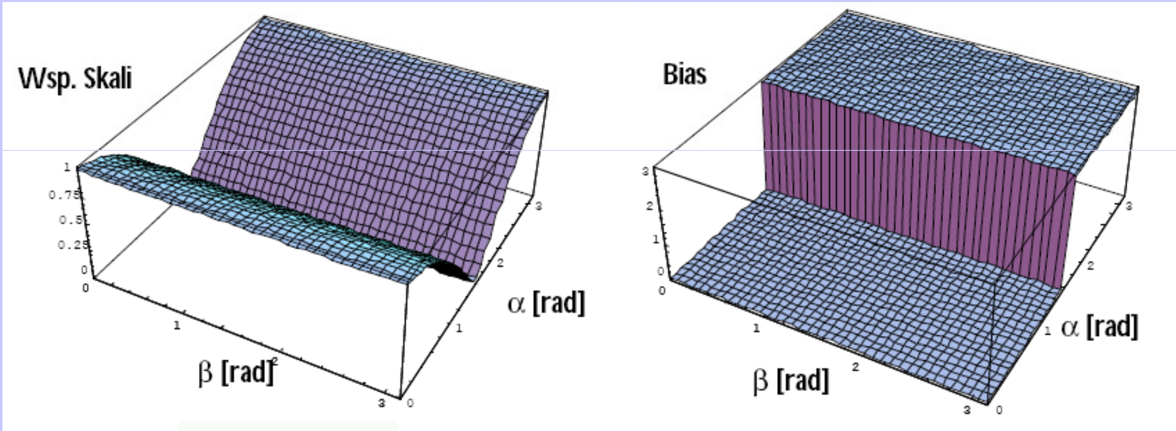
Mach-Zehnder interferometer:

$$m = \mathbf{E}^{+}_{we}\mathbf{R}(\Omega)\mathbf{G}(\delta)\mathbf{R}(\Phi)\mathbf{E}_{we}$$

a. Only twist: $\mathbf{M}_S = \mathbf{R}(\alpha)$

$$V = \sqrt{\cos^2\alpha + \sin^2\alpha \sin^2(2\beta)\sin^2\Delta}$$

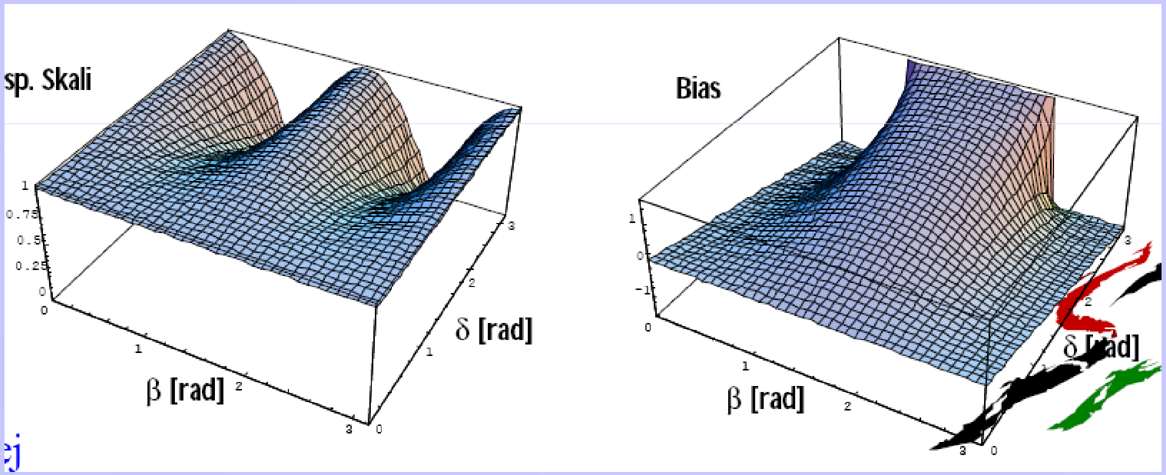
$$\phi_0 = arctg[-tg\alpha \sin(2\beta)\sin\Delta]$$



b. Only birefringence: $\mathbf{M}_S = \mathbf{G}(\delta)$

$$V = \sqrt{\cos^2(\delta/2) + \sin^2(\delta/2)\cos(2\beta)}$$

$$\phi_0 = arctg[-tg(\delta/2)\cos(2\beta)]$$



MZ (Ring Interferometer) is very sensitive on input SOP changes as well as polarisation properties of the optical fiber

Michelson (Fabry-Perot) interferometer:

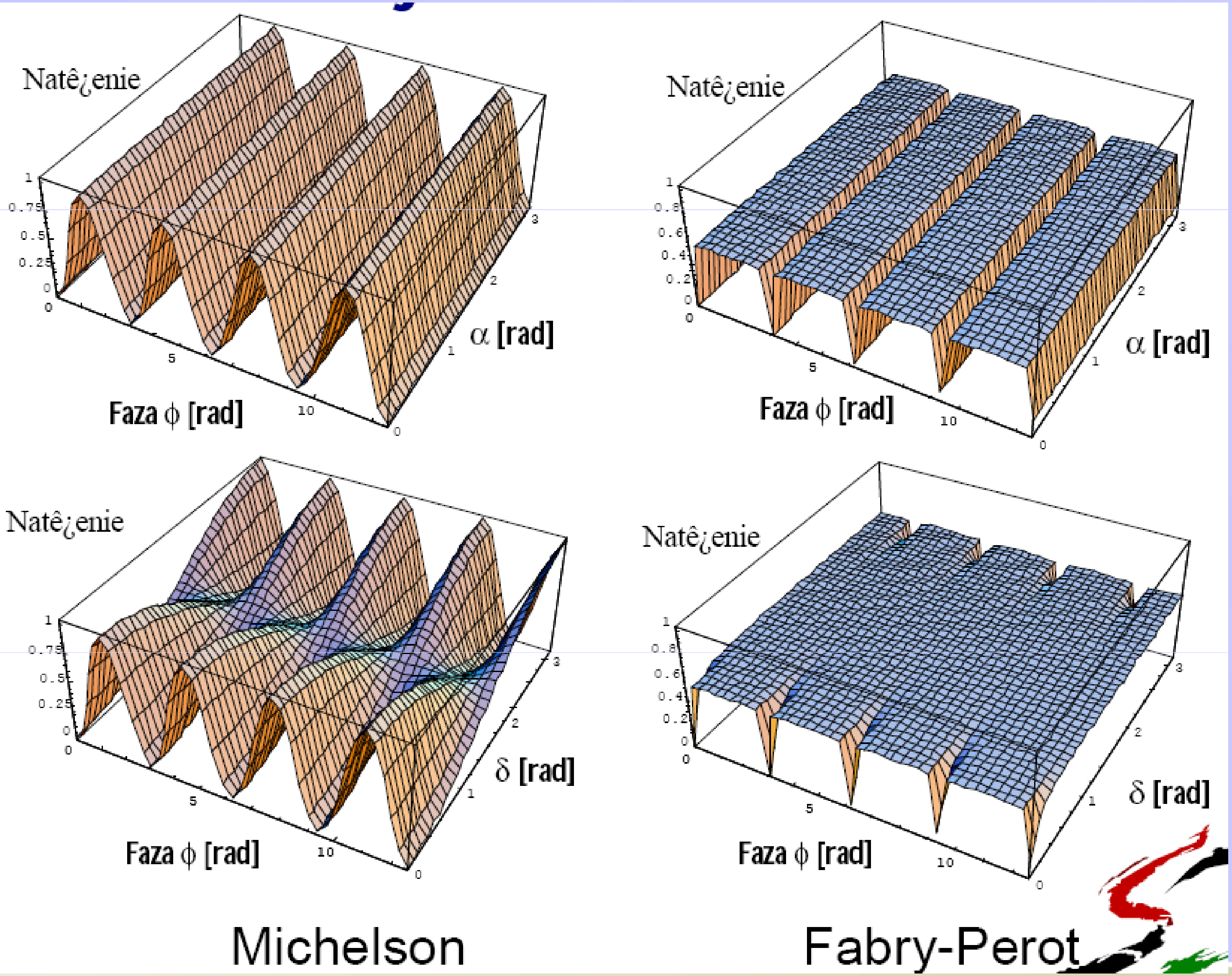
Input beam with linear SOP ($\beta=\pi/4$)

$$I = \frac{I_0}{1 + F\sin^2\phi/2}$$

$$F = 4R/(1 - R)^2$$

IM (FP) is more polarisation stabile.

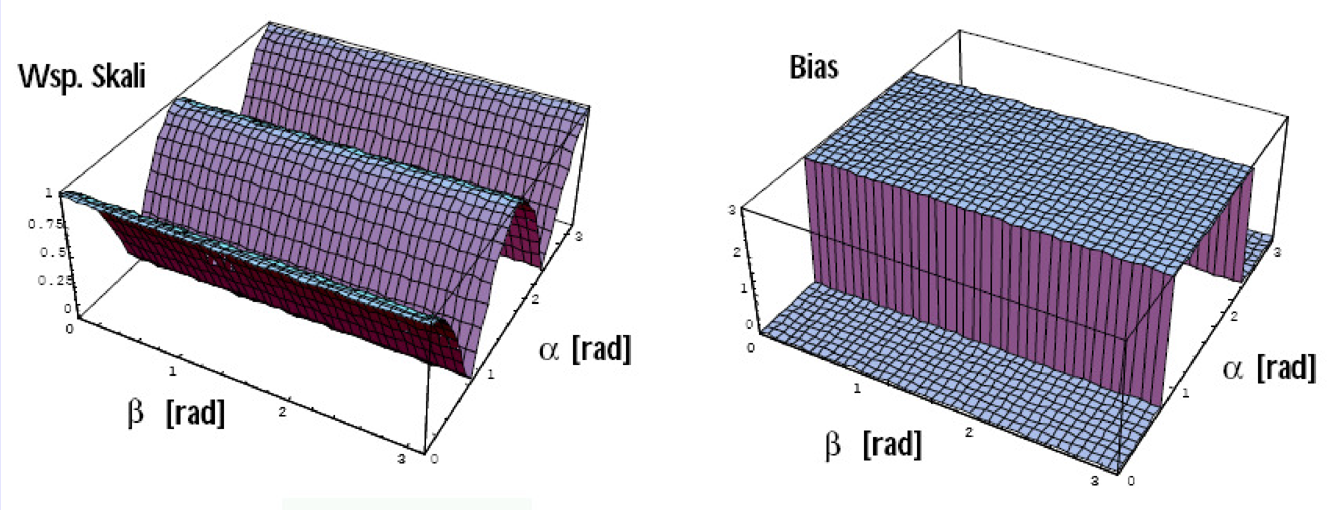
Noninfluence of fiber twist, but twice bigger influence of fiber birefringence.



Sagnac interferometer: $m = \mathbf{E}_{we}^{+}\mathbf{R}(\Omega)\mathbf{G}(-\delta)\mathbf{R}(\Phi+\Omega)\mathbf{G}(\delta)\mathbf{R}(\Phi)\mathbf{E}_{we}$

a. Only twist: $\mathbf{M}_S = \mathbf{R}(\alpha)$

$$V = \sqrt{\cos^2(2\alpha) + \sin^2(2\alpha)\sin^2(2\beta)\sin^2\Delta} \qquad \phi_0 = arctg[-tg(2\alpha)\sin(2\beta)\sin\Delta]$$



b. Only birefringence: $\mathbf{M}_S = \mathbf{G}(\delta)$

$$V = 1 \quad \phi_0 = 0$$

The Sagnac interferometer has nonsensitivity on clear birefringence of the optical fiber.

The polarisation influece reduction:

• application Hi-BI fiber (increase system cost 3 USD/m, the elements should be adjusted according SOP)

• polarisation correction via polarization controller

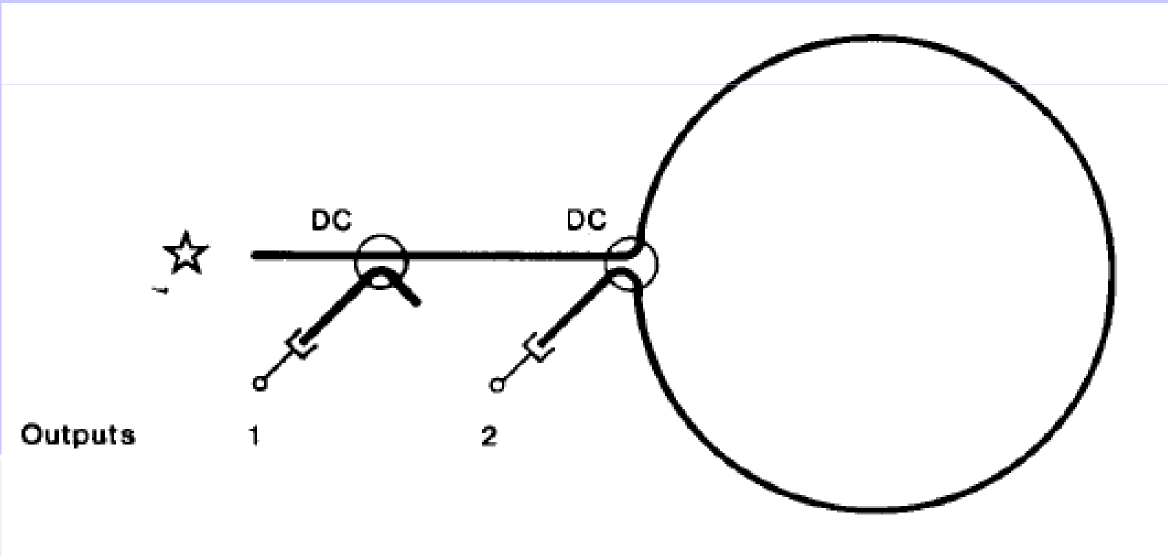
• reduction of the „freedom degrees" (Φ–δ–Ω):

  •differential – polarisation filtration (has been shown)

  • MZ and RR unstabile – active polarisation controller

  • choose the proper configuration:

    - CCW insted of CMW (more stabile)
    - SOP detection in real time (polarise-phase detection scheme)
    - reciprocal configuration of Sagnac interferometer



Dual wavelength interferometry (λ_1 i λ_2) – increase of the dynamic range

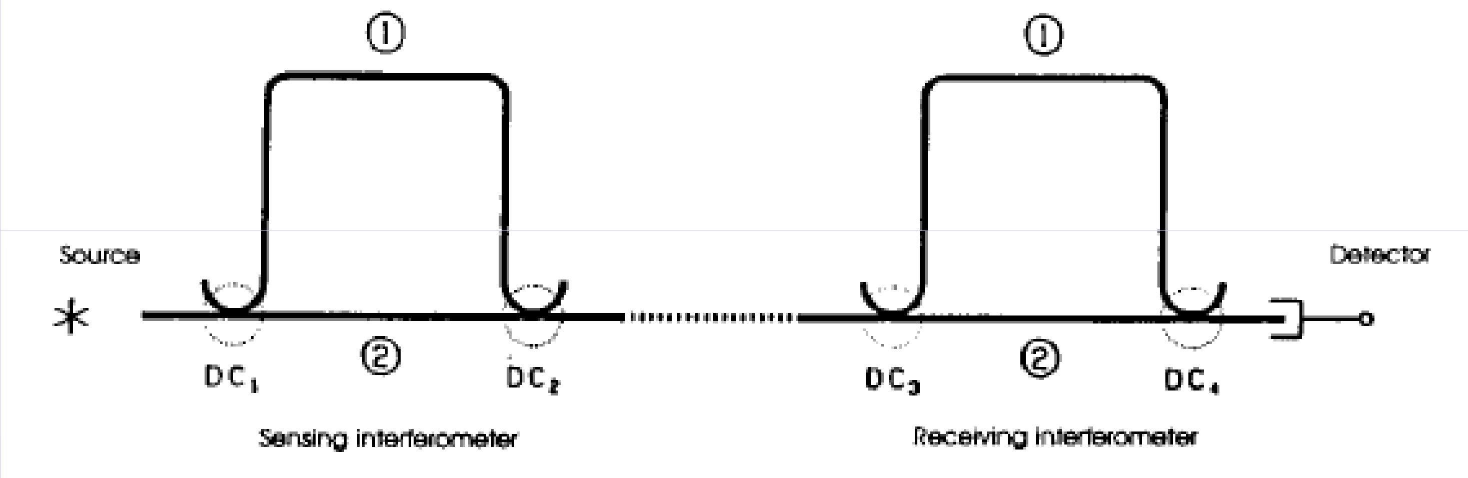
Transfer function:

$$I = I_0\left(1 + \cos\frac{2\pi nl}{\lambda_1}\right) + I_0\left((1 + \cos\frac{2\pi nl}{\lambda_2}\right)$$

$$I = I_0\left[1 + V\cos\left(\pi nl\,\frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2}\right)\right] \qquad V = \cos\left[\pi nl\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right)\right]$$

Increase of the dynamic range of the factor $\lambda_2/(\lambda_2 - \lambda_1)$

White light interferometer



An alternative technique for the extension of the dynamic range concerns the use of short coherence length source with gives possibility to measure phase as well as visibility.

Input electrical field:

$$\mathbf{E} = \mathbf{E}_{11} + \mathbf{E}_{12} + \mathbf{E}_{21} + \mathbf{E}_{22}$$

$$\mathbf{E}_{ij} = \tfrac{1}{4} E_0(\tau_{jk}) \exp i\omega(\tau_{jk})$$

$$\tau_{11} - \tau_{12} = \tau_{21} - \tau_{22} = \tau_a$$

$$\tau_{12} - \tau_{22} = \tau_{21} - \tau_{11} = \tau_b$$

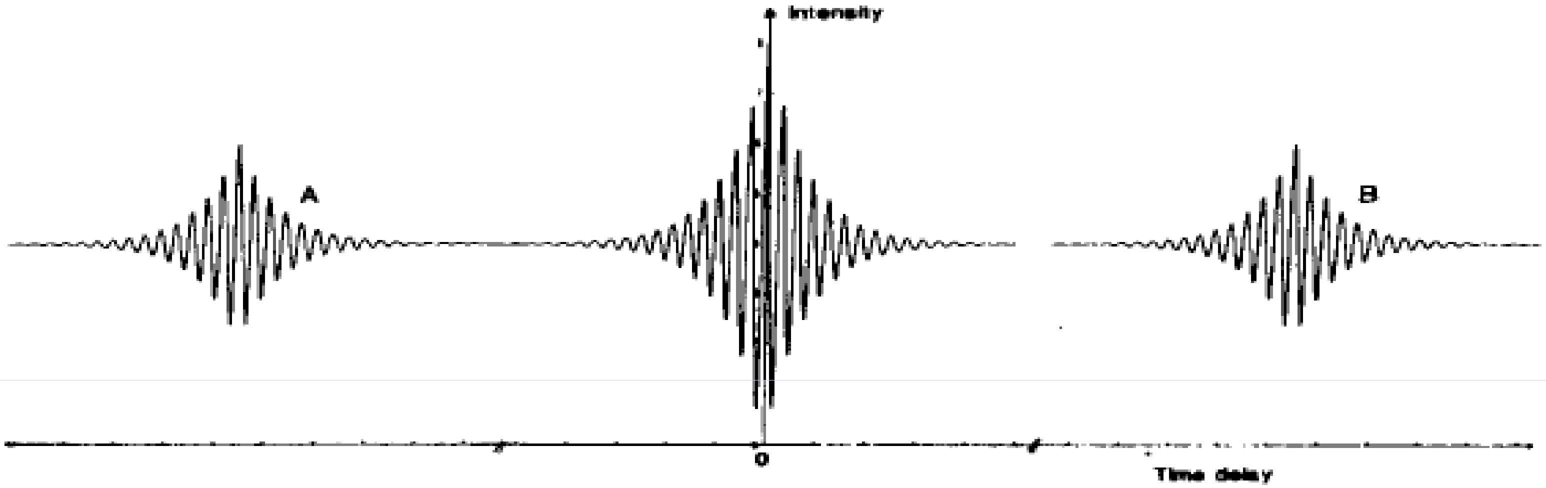
Describe time difference of ligth propagation between ways 1 and 2 for sensing and receiving interferometers => transfer function:

$\tau_b \gg \tau_c$

$$I = \tfrac{1}{4} I_0 \left[1 + \cos \omega \tau_a\right]$$

$\tau_a \approx \tau_b$

$$I \approx \tfrac{1}{4} I_0 \left[1 + \tfrac{1}{2} \cos \omega (\tau_a - \tau_b)\right]$$



Transfer Function of Tandem Interferometers

Fig. 10.6b Interference fringes produced by the arrangement shown Fig. 6a, as a function of path length imbalance (and hence time delay) in the receiving interferometer, where the path imbalance of the sensing interferometer is much greater than the coherence length of the source. The group of interference fringes around the origin correspond to near-zero imbalance of the receiving interferometer; groups A and B correspond to the receiving interferometer balancing the path difference in the sensing interferometer, and satisfy the conditions $\tau_a + \tau_b \lesssim \tau_c$ and $\tau_a - \tau_b \lesssim \tau_c$ respectively, where τ_a, τ_b and τ_c are as defined in the text.

Electronic considerations

Transfer function:

$$I_1 = I_0\,[1 - V\cos\,(\phi_a - \phi_b)]$$

Thus interferometer measures only relative phase delay between two arms instead of absolute phase delay.

The general problem of signal processing is recognize that part of phase changes which is correlated with measurand when exist different noise sources.

It is convenient to separate the measurand phase into a total contribution from noise sources (ϕ_d) and from a signal ($\phi_s \sin\omega_s t$).

$$\phi_a - \phi_b = \phi_d + \phi_s \sin\omega_s t$$

The photocurrent:

$$I_D = I_{OD}[1 + V\cos(\phi_d + \phi_s \sin\omega_s t)]$$

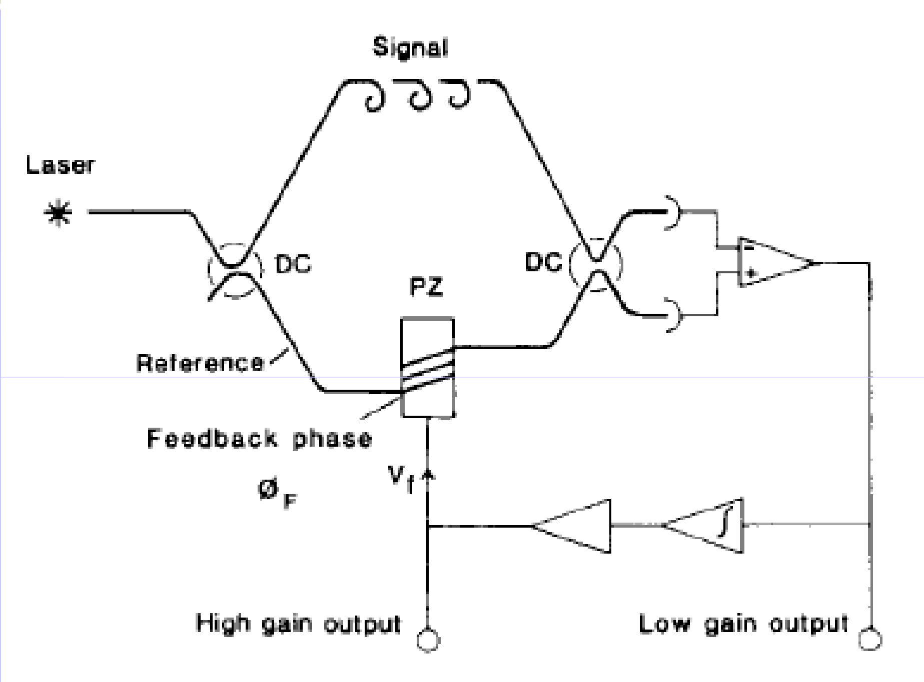
The frequency spectrum of the signal:

$$\cos(\phi_d + \phi_s \sin\omega_s t) = \cos\phi_d\{J_0(\phi_s) + 2\sum_{i=1}^{\infty} J_{2i}(\phi_s)\cos(2i\omega_s t)\} - \sin\phi_d\{2\sum_{i=0}^{\infty} J_{2i+1}(\phi_s)\sin[(2i+1)\omega_s t]\}$$

Because sensitivity depends from noise term ϕ_d the more sophisticated recovery technique must be applied for produce a useful signal.

Active homodyne technique

This scheme uses servo-element for taking interferometer in constant point of work i.e. (ϕ_d) = const. np. $\pi/2$ (work in quadrature)



Pasive homodyne technique

This scheme needs configuration with two output where exist constat phase difference – the best equal to $\pi/2$ radian. For example system with 3x3 output coupler or polarymetr with quaterwave plate:

$$I_{D1} \propto \cos[\phi_d + \phi_s \sin\omega_s t] \qquad I_{D2} \propto \sin[\phi_d + \phi_s \sin\omega_s t]$$

After applaying low-band filter

$$I_{D1} \propto \sin\phi_d J_1(\phi_s)\sin\omega_s t \qquad I_{D2} \propto \cos\phi_d J_1(\phi_s)\sin\omega_s t$$

After multiple and suming;

$$S \propto J_1(\phi_s)\sin\omega_s t \propto \phi_s \sin\omega_s t \qquad (\phi_s \ll 1)$$

Heterodyne techniques

Heterodyne processing protecs linear system response with infinite tracking range. *Heterodyne* means that the optical frequencies in the interferometer arms are unequal, and this is conventionally achieved using a frequency modulator such as a Bragg cell. Output takes the form

$$I_D \propto \cos(\omega_0 t + \phi_d + \phi_s \sin\omega_s t)$$

The output is thus a phase-modulated heterodyne carrier. The demdulation of such a siganl is a familiar electronic problem, and a number of techniques are established for its solution.